IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Dynamic correlations in domain growth: a 1/n expansion

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1990 J. Phys. A: Math. Gen. 23 4491

(http://iopscience.iop.org/0305-4470/23/20/011)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 129.252.86.83
The article was downloaded on 01/06/2010 at 09:21

Please note that terms and conditions apply.

J. Phys. A: Math. Gen. 23 (1990) 4491–4507. Printed in the UK

Dynamic correlations in domain growth: a $1/n$ expansion

T J Newman and A J Bray
Department of Theoretical Physics, The University, Manchester M13 9PL, UK

Received 13 June 1990

Abstract. We consider the dynamics of the n-component Ginzburg–Landau model with non-conserved order parameter (model A) following a quench from a high-temperature equilibrium state to zero temperature. The two-time correlation function of the order-parameter field is found in the $1/n$ expansion to have the asymptotic scaling form $C_{k}(t,t') = t'^{d/2}(t/t')^{\lambda/2}f(k^2t, k^2t')$ for $t \gg t'$, with $f(0,0) =$ constant. The form of the new exponent λ (which is a non-trivial function of n and d) was given explicitly to $O(1/n)$ in a recent letter. The purpose of this longer paper is to present a more detailed account of the calculation leading to the $O(1/n)$ form for λ. We also examine the role of thermal fluctuations in the ordered phase and the effect of long-range initial correlations on the ordering process.

1. Introduction

The process of non-equilibrium domain growth is a subject of intense interest [1]. Although many physical systems which undergo such a process are described by a scalar order parameter there has been much recent interest in the dynamics of domain growth of systems described by a vector order parameter [2–9]. In this paper we consider the dynamics of the n-component Ginzburg–Landau model following an instantaneous quench from a high-temperature equilibrium state to zero temperature. The central theme of the paper shall be a $1/n$ expansion. This is partly due to the lack of any other small parameter in which to develop a perturbation expansion, in contrast to working at the critical point where one may also develop an expansion about the upper critical dimension. We shall restrict our attention to the case where the order parameter is non-conserved (model A). In fact, systems with a conserved order parameter (model B) exhibit multiscaling following a zero temperature quench in the limit of $n \to \infty$ [6]. Our main finding is that correlations between the order parameter field at different times require a new, non-trivial exponent for their description.

The outline of this paper is as follows. In section 2 we shall define the model to be analysed and via a diagrammatic expansion derive the leading order results in the limit of $n \to \infty$ [2, 6, 8]. In section 3 we perform a $1/n$ expansion about the leading order results. This expansion gives rise to the main result of the paper which is that a new exponent arises in the description of the two-time correlation function $C_{k}(t,t') = [\phi^i_{k}(t)\phi^i_{-k}(t')]$ for late times. Here square brackets indicate an average over the ensemble of possible initial conditions, and $\phi_{k}(t)$ is simply the spatial Fourier transform of the vector order parameter field $\phi = (\phi^1, ..., \phi^n)$. The new exponent λ enters into the correlation function $C_{k}(t,t')$ in the scaling form [2, 3]:

$$C_{k}(t,t') = L(t')^d(L(t)/L(t'))^\lambda f(kL(t), kL(t')) \qquad t \gg t' \tag{1}$$

0305-4470/90/204491+17$03.50 © 1990 IOP Publishing Ltd

where $L(t) = t^{1/2}$ is the characteristic length scale of the system at time t and $f(0,0)$ is a constant. The above expression requires that both t and t' are large compared with some microscopic timescale t_0 (which will appear naturally in the following calculation). For $t' = 0$ the above relation still holds if we replace t' by t_0 on the RHS. λ has the following form:

$$\lambda = d/2 - (4/3)^{d/2}(2d(d+2)/9)B(d/2+1, d/2+1)(1/n) + \mathrm{O}(1/n^2) \tag{2}$$

where $B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the beta function [10]. We also present $1/n$ corrections to the scaling function $f(x,y)$.

Although all the results derived in this paper are calculated explicitly at zero temperature, one expects them to be valid throughout the ordered phase—the role of temperature should be limited to a renormalisation of the amplitudes for $T < T_c$. In section 4 we rederive the $n \to \infty$ results for general temperature and explicitly demonstrate the irrelevance of temperature in the ordered phase.

In section 5 we examine the effect of long-range initial correlations of the form $[\phi^i_{\boldsymbol{k}}(0)\phi^j_{-\boldsymbol{k}'}(0)] = (\Delta/k^\sigma)\delta_{i,j}\delta_{\boldsymbol{k},\boldsymbol{k}'}$ on the ordering process. A physically realizable situation of such long-range initial conditions would be a quench from the critical point into the low-temperature phase.

The paper concludes with a discussion of the results.

2. The model: $n \to \infty$ results

The dynamics governing the non-equilibrium relaxation of the quenched system are described by the Langevin equation

$$\partial\phi/\partial t = -\Gamma\delta\mathcal{H}(\phi)/\delta\phi \tag{3}$$

where $\mathcal{H}(\phi)$ has for example the Ginzburg–Landau form

$$\mathcal{H}(\phi) = \frac{1}{2}\int \mathrm{d}^d x\,\{-r\phi^2 + (\nabla\phi)^2 + (u/2n)(\phi^2)^2\}. \tag{4}$$

For model A dynamics we take $\Gamma = 1$, whereas $\Gamma = -\nabla^2$ for the case of model B dynamics. Notice that there is no noise term on the RHS of equation (3) since we envisage a quench to zero temperature. Therefore all averaging will be taken over the ensemble of initial conditions. We take for convenience the distribution of initial conditions to be Gaussian with zero mean and correlator defined (in terms of Fourier components of ϕ) by

$$[\phi^i_{\boldsymbol{k}}(0)\phi^j_{-\boldsymbol{k}'}(0)] = \Delta\delta_{i,j}\delta_{\boldsymbol{k},\boldsymbol{k}'}. \tag{5}$$

With the form of Hamiltonian given above, the equation of motion in terms of Fourier components of ϕ reads as follows

$$\partial\phi^i_{\boldsymbol{k}}/\partial t = (r-k^2)\phi^i_{\boldsymbol{k}} - (u/n)\sum_{\boldsymbol{p},\boldsymbol{q},j}\phi^j_{\boldsymbol{p}}\phi^j_{\boldsymbol{q}}\phi^i_{\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}}. \tag{6}$$

Physical quantities of interest to be obtained from this model are the response function

$$G_{\boldsymbol{k}}(t) = \left[\frac{\partial \phi^i_{\boldsymbol{k}}(t)}{\partial \phi^i_{\boldsymbol{k}}(0)}\right] \tag{7}$$

the two-time correlation function

$$C_{\boldsymbol{k}}(t,t') = [\phi^i_{\boldsymbol{k}}(t)\phi^i_{-\boldsymbol{k}}(t')] \tag{8}$$

and the energy

$$E(t) = [\mathcal{H}]. \tag{9}$$

The structure factor, $S_{\boldsymbol{k}}(t)$ is simply given by $C_{\boldsymbol{k}}(t,t)$. As is well known, $S_{\boldsymbol{k}}(t)$ often has an asymptotic scaling form [1]

$$S_{\boldsymbol{k}}(t) = L(t)^d g(kL(t)) \tag{10}$$

where $L(t) \sim t^{1/z}$ is the characteristic length scale in the system at time t (z is the dynamic exponent at the zero temperature fixed point [7]) and $g(x)$ is a scaling function. We should also note that $G_{\boldsymbol{k}}(t)$ is trivially related via integration by parts to $C_{\boldsymbol{k}}(t,0)$, i.e. $G_{\boldsymbol{k}}(t) = C_{\boldsymbol{k}}(t,0)/\Delta$ (for the case of Gaussian initial conditions).

At present there is no available method for solving (6) for general n. Therefore we develop a perturbation expansion about the trivially solved Gaussian model ($u = 0$) in powers of u. In this section we shall derive expressions for the above quantities of interest in the limit of $n \to \infty$. Setting $u = 0$ in the equation of motion yields

$$\phi^i_{\boldsymbol{k}}(t)\,|_{u=0} = \phi^i_{\boldsymbol{k}}(0)\exp(r - k^2)t \equiv \phi^i_{\boldsymbol{k}}(0)g_{\boldsymbol{k}}(t). \tag{11}$$

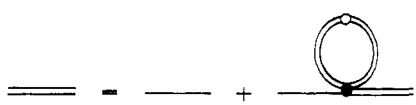

Figure 1. Diagrammatic form of the self-consistent equation (12) for the response function in the limit of $n \to \infty$.

We now construct $\phi^i_{\boldsymbol{k}}(t)$ as an expansion about this result in powers of u. It is convenient to represent the expansion in terms of diagrams [11] whose rules are easily derived from studying explicitly the first few terms in the expansion. To calculate $G_{\boldsymbol{k}}(t)$ we differentiate the expansion with respect to $\phi^i_{\boldsymbol{k}}(0)$ and average over the ensemble of initial conditions (whose distribution is defined above). The reason for the $n \to \infty$ limit being a soluble case is the following. For each diagram we can attribute a definite order in terms of powers of $1/n$. Each vertex carries a factor of u/n and each closed loop is freely summed over the relevant spin component, therefore contributing a factor of n. So a diagram with v vertices and c closed loop is of $O(n^{c-v})$. In taking the limit of $n \to \infty$ only diagrams with the maximal number of closed loops in any given order of u/n will survive. We have therefore reduced the infinitely diverse set of diagrams to a much simpler subset which may be resummed to give a self-consistent expression for the response function. In figure 1 we show the Dyson-like equation which represents the self-consistent equation for the response function. The circle represents the initial correlation of fields (see (5)), the single lines represent $g_{\boldsymbol{k}}(t)$ and the double lines represent the full response function (in the limit $n \to \infty$) $G_{\boldsymbol{k}}(t)$. Figure 1 corresponds to the equation

$$G_{\boldsymbol{k}}(t) = g_{\boldsymbol{k}}(t) - u\Delta \int_0^t \mathrm{d}t'\, g_{\boldsymbol{k}}(t-t') G_{\boldsymbol{k}}(t') \sum_{\boldsymbol{p}} G_{\boldsymbol{p}}(t') G_{-\boldsymbol{p}}(t'). \tag{12}$$

Differentiating with respect to t yields a first-order differential equation for $G_{\boldsymbol{k}}(t)$ whose solution may be written in the following form:

$$G_{\boldsymbol{k}}(t) = \exp\left(- \int_0^t \mathrm{d}t' \,\{k^2 + A(t')\}\right) \tag{13}$$

where

$$A(t) = -r + u\Delta \sum_{\boldsymbol{p}} G_{\boldsymbol{p}}(t)^2. \tag{14}$$

To determine $A(t)$ we substitute (13) into (14) and perform an unrestricted sum over the momentum $\boldsymbol{p}$. This yields

$$A(t) = -r + uK_d \Delta t^{-d/2} \exp\left(-2 \int_0^t \mathrm{d}t' A(t') \right) \tag{15}$$

where K_d is defined by $\sum_{\boldsymbol{p}} \exp(-2p^2 t) = K_d / t^{d/2}$. In section 4 we shall study an equation similar to this in detail in order to determine the precise role of thermal fluctuations in the ordering process (following a quench to finite temperature within the ordered phase). However for the equation above we will concentrate on the asymptotic behaviour of $A(t)$ (i.e when $rt \gg 1$). It is clear that we require $A(t) \to -d/4t$ for $t \to \infty$ for consistency. This implies $\int_0^t \mathrm{d}t' A(t') \to -(d/4)\ln(t/t_0)$ for large t. We have introduced the short-time cut-off t_0 which characterises the time at which the above asymptotic behaviour becomes valid. Determining t_0 self-consistently from (15) gives

$$r = uK_d \Delta / t_0^{d/2}. \tag{16}$$

We now have the $n \to \infty$ form for the response function

$$G_{\boldsymbol{k}}(t) = (t/t_0)^{d/4} \exp(-k^2 t). \tag{17}$$

At leading order the two-time correlation function is trivially related to the response function via $C_{\boldsymbol{k}}(t,t') = \Delta G_{\boldsymbol{k}}(t) G_{-\boldsymbol{k}}(t')$. We therefore have

$$C_{\boldsymbol{k}}(t,t') = \Delta (tt'/t_0^2)^{d/4} \exp\{-k^2(t+t')\} \tag{18}$$

for $t, t' \gg t_0$. Setting $t' = t$ in (18) reveals that the structure function $S_{\boldsymbol{k}}(t)$ has the expected scaling form (10) at leading order with $L(t) = t^{1/2}$ and $g(x) = (r/uK_d)\exp(-2x)$. Also we notice that as expected $\sum_{\boldsymbol{k}} S_{\boldsymbol{k}}(t) = r/u$ equal to the square of the equilibrium magnetisation.

It remains in this section to calculate the energy $E(t)$ to leading order (i.e. for $n \to \infty$). The averaged energy per degree of freedom is given by

$$\epsilon(t) \equiv E(t)/n = (1/2n)\left(\int \mathrm{d}^d x\, \{(\nabla\phi)^2 - r\phi^2 + (u/2n)(\phi^2)^2\}\right). \tag{19}$$

To evaluate $\epsilon(t)$ we express the RHS of the above equation in terms of the Fourier transformed fields $\phi^i_{\boldsymbol{k}}(t)$ to give (to leading order)

$$\epsilon(t) = (1/2)\sum_{\boldsymbol{k}}(k^2 - r)S_{\boldsymbol{k}}(t) + (u/4)\sum_{\boldsymbol{k},\boldsymbol{p}} S_{\boldsymbol{p}}(t)S_{\boldsymbol{k}}(t) \tag{20}$$

where $S_{\boldsymbol{k}}(t)$ is the structure function evaluated above for $n \to \infty$. Explicit evaluation of (20) then gives

$$\epsilon(t) = -(r^2/4u) + (r/u)(d/8t) \tag{21}$$

where the first term is the trivial condensation energy. The second term indicates that the energy of the system relaxes to equilibrium as $1/t$. This form of energy relaxation is suggested by dimensional analysis [9] and has been confirmed in a recent series of numerical simulations [9]. These simulations were based on a 'hard-spin' model (where the vector field is constrained to have magnitude $\sqrt{n}$) which corresponds to taking $r \to \infty$ and $u \to \infty$ with $r/u = 1$. In fact one may rederive all the $n \to \infty$ results directly from an equation of motion derived explicitly for the 'hard-spin' case [9]. As can be seen above (and in what follows) the parameters r and u appear in the ratio r/u in all results of interest implying that the 'soft-spin' and the 'hard-spin' models are in the same universality class.

We have now presented the leading order results for the quantities of interest. In the next section we shall see that extending the calculation to $\mathrm{O}(1/n)$ yields new and unexpected behaviour for the two-time correlation function $C_{\boldsymbol{k}}(t, t')$.

3. $1/n$ expansion: new results

To extend the results of the previous section requires analysing the $\mathrm{O}(1/n)$ terms in the diagrammatic expansion. Relevant diagrams will therefore have v vertices and $c = v - 1$ closed loops thus contributing to $\mathrm{O}(n^{c-v}) = \mathrm{O}(1/n)$. Writing

$$G_{\boldsymbol{k}}(t) = G^{\infty}_{\boldsymbol{k}}(t) + G'_{\boldsymbol{k}}(t)/n + \mathrm{O}(1/n^2) \tag{22}$$

and

$$C_{\boldsymbol{k}}(t, t') = C^{\infty}_{\boldsymbol{k}}(t, t') + C'_{\boldsymbol{k}}(t, t')/n + \mathrm{O}(1/n^2) \tag{23}$$

we find that $G'_{\boldsymbol{k}}(t)$ and $C'_{\boldsymbol{k}}(t, t')$ may be expressed in the following way:

$$G'_{\boldsymbol{k}}(t) = 2\int_0^t \mathrm{d}t_1 \int_0^{t_1} \mathrm{d}t_2 G^{\infty}_{\boldsymbol{k}}(t_2) G^{\infty}_{\boldsymbol{k}}(t, t_1)\Pi_{\boldsymbol{k}}(t_1, t_2) \tag{24}$$

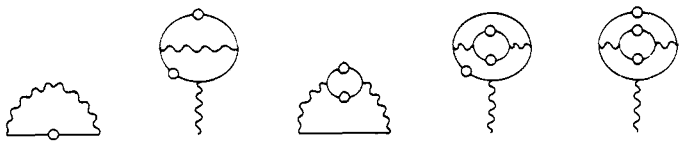

Figure 2. Diagrams contributing to the 'self-energy' $\Pi_{\boldsymbol{k}}(t_1, t_2)$ at $O(1/n)$. Diagrams (b) and (d) each carry a combinatoric factor of 2.

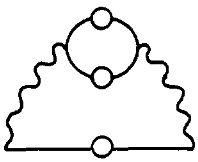

Figure 3. $O(1/n)$ diagram for the function $\Omega_{\boldsymbol{k}}(t_1, t_2)$ of (25).

and

$$C'_{\boldsymbol{k}}(t,t') = \Delta\{G^{\infty}_{\boldsymbol{k}}(t)G'_{\boldsymbol{k}}(t') + G^{\infty}_{\boldsymbol{k}}(t')G'_{\boldsymbol{k}}(t)\} + 2\int_0^t \mathrm{d}t_1 \int_0^{t'} \mathrm{d}t_2 G^{\infty}_{\boldsymbol{k}}(t,t_1)G^{\infty}_{\boldsymbol{k}}(t',t_2)\Omega_{\boldsymbol{k}}(t_1,t_2). \tag{25}$$

The functions $\Pi_{\boldsymbol{k}}(t_1, t_2)$ and $\Omega_{\boldsymbol{k}}(t_1, t_2)$ are expressed in terms of diagrams and are shown in figures 2 and 3 respectively.

The elements of the diagrams are as follows. A circle represents the zero-time correlation of two fields and therefore carries a weight (see (5)) $\Delta\delta_{i,j}\delta_{\boldsymbol{k},\boldsymbol{k}'}$. A single line emerging from a circle represents the response function calculated to leading order, $G^{\infty}_{\boldsymbol{k}}(t)$ (see (17)). The point at which it terminates then corresponds to a time t. There are two further elements appearing in the diagrams which did not appear in the leading order calculation. The first of these is a single line connecting two non-zero times t, t' where $t > t'$. This is written as $G^{\infty}_{\boldsymbol{k}}(t,t')$ and is the response of a field at time t to thermal noise acting at time t' in the limit of infinitesimal noise. Therefore we have

$$G^{\infty}_{\boldsymbol{k}}(t,t') = \left[\frac{\delta\phi^i_{\boldsymbol{k}}(t)}{\delta\xi^i_{\boldsymbol{k}}(t')}\right]_{\xi=0}. \tag{26}$$

This is easily calculated by following the derivation of $G^{\infty}_{\boldsymbol{k}}(t)$ in section 1 but retaining a lower limit in the time integrals equal to t' rather than zero. Explicit calculation yields

$$G^{\infty}_{\boldsymbol{k}}(t,t') = \frac{G^{\infty}_{\boldsymbol{k}}(t)}{G^{\infty}_{\boldsymbol{k}}(t')} = (t/t')^{d/4}\exp\{-k^2(t-t')\}. \tag{27}$$

The second new element is the wavy line, $v_{\boldsymbol{k}}(t,t')$, that appears in the diagrams connecting two times t, t' with $t > t'$. This corresponds to the 'dressed' vertex. By writing down the usual 'bubble sum' of the $1/n$ expansion we can derive the self-consistent equation for the wavy line illustrated in figure 4. This diagrammatic equation corresponds to

$$v_{\boldsymbol{k}}(t,t') = u\delta(t-t') - 2u\Delta \int_{t'}^{t} \mathrm{d}t''\, v_{\boldsymbol{k}}(t,t'') \sum_{\boldsymbol{p}} G_{\boldsymbol{p}}^{\infty}(t') G_{-\boldsymbol{p}}^{\infty}(t'') G_{\boldsymbol{k}+\boldsymbol{p}}^{\infty}(t'',t'). \tag{28}$$

By substituting the form of the functions $G_{\boldsymbol{k}}^{\infty}(t)$ and $G_{\boldsymbol{k}}^{\infty}(t,t')$ into (28) we may easily evaluate the momentum sum over $\boldsymbol{p}$. Making use of (16) then yields the following integral equation for $v_{\boldsymbol{k}}(t,t')$.

$$v_{\boldsymbol{k}}(t,t') = u\delta(t-t') - 2r \int_{t'}^{t} \mathrm{d}t''\, v_{\boldsymbol{k}}(t,t'') \exp\left(-k^2 \frac{(t''^2 - t'^2)}{2t''}\right). \tag{29}$$

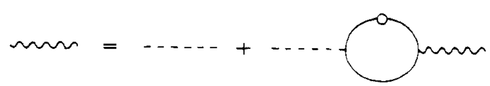

Figure 4. Self-consistent equation for the wavy line (i.e. $v_{\boldsymbol{k}}(t_1,t_2)$) which corresponds to (28).

The solution of (29) for $v_{\boldsymbol{k}}(t,t')$ constitutes the most difficult part of this $1/n$ calculation. In fact we have been unable to solve this integral equation in closed form. However since we are only interested in the asymptotic behaviour of the system, the lack of an exact solution of (29) need not bar us from completing the calculation. Progress is made by developing $v_{\boldsymbol{k}}(t,t')$ as a controlled expansion about the solution of a simpler, soluble integral equation. We construct an integral equation for some function $f_{\boldsymbol{k}}(t,t')$ that mirrors (29) as closely as possible, but where the kernel has the property $K(t'',t') = K^*(t''-t')$. We can solve such an integral equation by Laplace transform methods, and we are then in a position to write down an expansion for $v_{\boldsymbol{k}}(t,t')$ about the function $f_{\boldsymbol{k}}(t,t')$.

We choose as our soluble integral equation

$$f_{\boldsymbol{k}}(t,t') = u\delta(t-t') - 2r \int_{t'}^{t} \mathrm{d}t''\, f_{\boldsymbol{k}}(t,t'') \exp\{-k^2(t''-t')\}. \tag{30}$$

The solution of this equation is easily obtained (by Laplace transformation) to give

$$f_{\boldsymbol{k}}(t,t') = u\{\delta(t-t') - 2r\exp[-2r(t-t')]\}\ \exp\{-k^2(t-t')\}. \tag{31}$$

Writing

$$v_{\boldsymbol{k}}(t,t') = f_{\boldsymbol{k}}(t,t')\rho_{\boldsymbol{k}}(t,t') \tag{32}$$

and substituting this into (29) enables us to develop $\rho_{\boldsymbol{k}}(t,t')$ as an expansion of terms involving the form $(t-t')^2/t$. Explicitly one finds

$$\rho_{\boldsymbol{k}}(t,t') = 1 + \frac{k^2(t-t')^2}{2t}[1 + \mathrm{O}(r(t-t'))] \tag{33}$$

plus terms of higher order in $k^2(t-t')^2/t$. The key property of $\rho_{\boldsymbol{k}}(t,t')$ is that the next to leading term is quadratic in the time difference. This enables us to set $\rho_{\boldsymbol{k}}(t,t') = 1$ when we are evaluating the above diagrams in the asymptotic regime $rt \gg 1$. To illustrate this consider integrating over the time arguments of the wavy line, i.e. integrating $v_{\boldsymbol{k}}(t,t')h(t')$ over t' for some general function $h(t')$. One then finds via integration by parts the following relation:

$$\int^t \mathrm{d}t' v_{\boldsymbol{k}}(t,t')h(t') = \frac{1}{2r}\left(\frac{\partial}{\partial t'}[\exp -k^2(t-t')]\rho_{\boldsymbol{k}}(t,t')h(t')\right)_{t'=t}\left(1+\mathrm{O}\left(\frac{1}{rt}\right)\right). \quad (34)$$

This implies that for all leading-order results we may simply set $\rho_{\boldsymbol{k}}(t,t') = 1$, because $\partial\rho_{\boldsymbol{k}}(t,t')/\partial t'_{t'=t} = 0$.

We now have all the necessary elements to calculate the diagrams shown in figures 2 and 3. If one is solely interested in evaluating the new exponent λ it is convenient to set the external momentum $\boldsymbol{k} = 0$ when computing the above diagrams. The results of such a calculation were presented in [2]. In this paper, however, we present results for the $1/n$ corrections to the response function and the two-time correlation function for general $\boldsymbol{k}$.

Given the above 'ingredients', evaluation of the diagrams is straightforward but tedious. For this reason we shall only present the final results for evaluation of the diagrams. Referring back to (22)–(25) we find that

$$G_{\boldsymbol{k}}(t) = G_{\boldsymbol{k}}^{\infty}(t)\left(1 - \frac{(4/3)^{d/2}}{16n}\left[\Lambda_1(t) + 2\Lambda_2(t) - 2\Lambda_3(t,k^2t)\right] + \mathrm{O}(1/n^2)\right) \quad (35)$$

and

$$C_{\boldsymbol{k}}(t,t') = C_{\boldsymbol{k}}^{\infty}(t,t')\Bigg(1 + \frac{(4/3)^{d/2}}{16n}\big[2\Upsilon(t',k^2t') - \Lambda_1(t) - \Lambda_1(t') - 2(\Lambda_2(t) + \Lambda_2(t') - \Lambda_3(t,k^2t) - \Lambda_3(t',k^2t'))\big] + \mathrm{O}(1/n^2)\Bigg). \quad (36)$$

With reference to figure 2: diagrams (a) and (b) cancel exactly in the asymptotic regime when integrated over the external propagators; Λ_1, Λ_2, and Λ_3 are the contributions from diagrams (e), (d) and (c) respectively. Υ is the contribution from the diagram shown in figure 3. Since the functions Λ_i and Υ have a rather complex form we present them in full in the appendix.

To analyse the functions in the appendix we examine the powers of y that appear in the integrands. We see that each function has a leading contribution of $\ln(t/t_0)$. However, the logarithmic contributions from Λ_2 and Λ_3 are identical which implies

$$\Lambda_2 - \Lambda_3 \sim \mathrm{O}(k^2t). \quad (37)$$

Therefore there is only one logarithmic contribution to the response function. Evaluating this contribution (from Λ_1) explicitly we assume that we may exponentiate the logarithm to find

$$G_{\boldsymbol{k}}(t) = (t/t_0)^{\lambda/2}h(k^2t) \quad (38)$$

where λ is given by (2) and the scaling function $h(x)$ is the leading-order result ($\exp\{-x\}$) together with $1/n$ contributions from the Λ functions. It is important to note that the scaling variable k^2t remains unchanged to $O(1/n)$ (and presumably to all further orders) indicating that the dynamic exponent $z = 2$ independent of n.

The same analysis applies to the two-time correlation function. Now there are logarithmic contributions from Υ as well as from Λ_1. Therefore we have (for $t \gg t'$)

$$C_{\boldsymbol{k}}(t,t') = C^{\infty}_{\boldsymbol{k}}(t,t')\{1 + (a/n)[\ln(t'/t_0) + \ln(t/t_0) - 2\ln(t'/t_0)] + (1/n)F(k^2t, k^2t') + O(1/n^2)\} \tag{39}$$

where a is simply the $1/n$ contribution to $\lambda/2$. Since $L(t) = t^{1/2}$, the above equation may be cast into the form given by (1) (where we have again assumed that we may exponentiate the logarithms).

It turns out that the structure factor $S_{\boldsymbol{k}}(t)$ ($= C_{\boldsymbol{k}}(t,t)$) to $O(1/n)$ is given by (39) with $t = t'$. We see that the logarithms in (39) will cancel for this case and this implies that the structure factor retains its standard scaling form (see (10)) to $O(1/n)$ and presumably to all further orders. The scaling function of the structure function does however pick up $1/n$ corrections as expected. Writing $S_{\boldsymbol{k}}(t) = (r/u)(8\pi t)^{d/2}g(k^2t)$ we may express the scaling function in the form

$$g(x) = g^{\infty}(x) + (1/n)g'(x) + O(1/n^2) \tag{40}$$

where $g^{\infty}(x) = \exp(-2x)$ as evaluated in section 2. By numerically evaluating the functions Λ_i and Υ we can find the form of $g'(x)$ for various values of d. In figure 5 we present the form of $g'(x)$ for $d = 1, 2$ and 3.

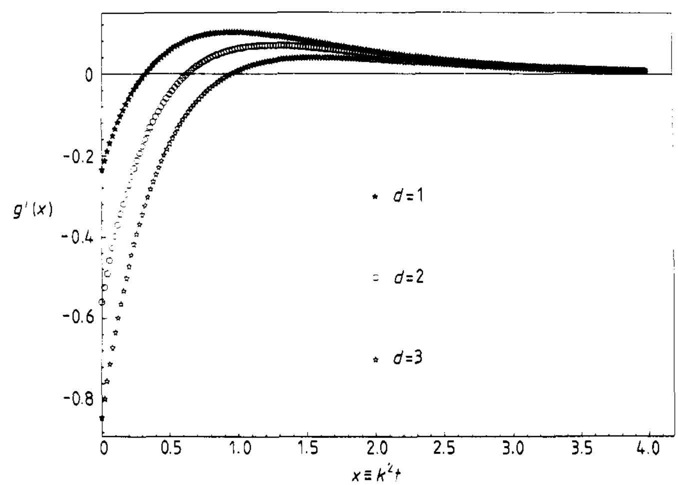

Figure 5. $O(1/n)$ corrections to the scaling function of the structure factor as defined in (40) for $d = 1, 2$ and 3.

A nice check on the correctness of the form of the Λ and Υ is the following. We know that summing the structure factor over all $\boldsymbol{k}$ must yield the square of the equilibrium magnetization ($= r/u$). Since this was found to hold for the leading order form of $S_{\boldsymbol{k}}(t)$ (see section 2) we must then have

$$\sum_{\boldsymbol{k}} S'_{\boldsymbol{k}}(t) = 0 \tag{41}$$

where $S'_{\boldsymbol{k}}(t)$ is the $1/n$ correction to the structure factor. The proof of this is tedious but this stringent condition has been verified using the functions Λ_i and Υ given in the appendix.

We end this section by discussing numerical simulations relevant to the above results. A recent series of simulations [9] measured the autocorrelation function $A(t) = [\phi(\boldsymbol{x},t)\phi(\boldsymbol{x},0)]$, i.e. the correlation with the initial condition, for model A domain growth in vector spin systems for various values of n in $d = 1$. Using $A(t) = \int \mathrm{d}^d k\, C_{\boldsymbol{k}}(t,0)$, the relation $C_{\boldsymbol{k}}(t,0) = \Delta G_{\boldsymbol{k}}(t)$, and the scaling form (38) yields $A(t) \sim t^{-(d-\lambda)/2}$. For $d = 1$ we expect the above analysis to correctly describe the simulations for $n \geq 3$ since the case of $n = 2$ is found (by simple analytic arguments [9]) to be anomalous. The simulations verified that λ is indeed dependent on n and comparison of the measured values of λ with the result given in (2) is surprisingly good. Results of similar simulations for $d = 2$ will be presented shortly [12]. Again the results for λ are in good agreement with (2) for $n \geq 4$. It has been found, however, that topological structures not present for large n play an important role in the dynamics of $d = 2$ systems with $n = 2$ and 3. Further discussion of these more complex systems will be deferred to a future publication [12].

4. The role of thermal fluctuations for $T < T_c$

The above analysis describes a system quenched from a high temperature equilibrium state to zero temperature. To what extent are the results obtained valid for the more general case of a quench from high temperature to some finite temperature in the ordered phase? To answer this question we must restrict our attention to the case of $d > 2$ ($d > 1$ for a scalar order parameter) where a low-temperature ordered phase exists. We shall consider only the limit $n \to \infty$.

A quench to finite temperature necessitates the inclusion of a noise term in the Langevin equation. Therefore we have

$$\partial\phi^i_{\boldsymbol{k}}/\partial t = (r - k^2)\phi^i_{\boldsymbol{k}} - (u/n)\sum_{j,\boldsymbol{p},\boldsymbol{q}} \phi^i_{\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}}\phi^j_{\boldsymbol{p}}\phi^j_{\boldsymbol{q}} + \xi^i_{\boldsymbol{k}}(t) \tag{42}$$

where the noise is taken to have a Gaussian distribution with zero mean and correlator given by

$$\langle \xi^i_{\boldsymbol{k}}(t)\xi^j_{-\boldsymbol{k}'}(t')\rangle = 2T\delta_{i,j}\delta_{\boldsymbol{k},\boldsymbol{k}'}\delta(t-t'). \tag{43}$$

Notice that we denote averages over the thermal noise by angled brackets and we shall continue to denote averages over the ensemble of initial conditions by square brackets. Let us calculate the effect of thermal fluctuations on the response function defined by

$$G_{\boldsymbol{k}}(t) = \left\langle \left[\frac{\partial\phi^i_{\boldsymbol{k}}(t)}{\partial\phi^i_{\boldsymbol{k}}(0)}\right]\right\rangle. \tag{44}$$

By an identical diagrammatic analysis to that described in section 2 we may derive a self-consistent equation for $G_{\boldsymbol{k}}(t)$. We find

$$G_{\boldsymbol{k}}(t) = g_{\boldsymbol{k}}(t) - u\int_0^t \mathrm{d}t'\, g_{\boldsymbol{k}}(t-t')G_{\boldsymbol{k}}(t')\sum_{\boldsymbol{p}} G_{\boldsymbol{p}}(t')^2 \left(\Delta + 2T\int_0^{t'} \mathrm{d}t''\, G_{\boldsymbol{p}}(t'')^{-2}\right). \tag{45}$$

This may be written as (see section 2)

$$G_{\boldsymbol{k}}(t) = \exp\left(-\int_0^t \mathrm{d}t'(k^2 + A(t'))\right) \tag{46}$$

where

$$A(t) + r = u\sum_{\boldsymbol{p}} G_{\boldsymbol{p}}(t)^2\left(\Delta + 2T\int_0^t \mathrm{d}t'\, G_{\boldsymbol{p}}(t')^{-2}\right). \tag{47}$$

Substituting (46) into (47) enables us to write a self-consistent equation for $A(t)$. It is convenient, however, to work instead with the function $\eta(t)$, defined by

$$\eta(t) \equiv \exp\left(2\int_0^t \mathrm{d}t'\, A(t')\right) \tag{48}$$

which satisfies the self-consistent equation

$$\dot{\eta}(t) + 2r\eta(t) = 2u\sum_{p}\exp(-2p^2 t)\left(\Delta + 2T\int_0^t \mathrm{d}t'\,\eta(t')\exp(2p^2 t')\right). \tag{49}$$

This integro-differential equation may be solved by Laplace transformation methods. Defining $\tilde{\eta}(s)$ as the Laplace transform of $\eta(t)$ we may solve the above equation to find

$$\tilde{\eta}(s) = \frac{2u\Delta\mathcal{J} + 1}{s + 2r - 4uT\mathcal{J}} \tag{50}$$

where

$$\mathcal{J} = \mathcal{L}\left(\sum_{p}\exp(-2p^2 t)\right) \tag{51}$$

($\mathcal{L}[x(t)]$ indicates the Laplace transform of $x(t)$). Explicit evaluation of $\mathcal{J}$ (with the inclusion of a high momentum cut-off k_m) yields (for the range $2 < d < 4$)

$$\tilde{\eta}(s) = \frac{c_1 - \Delta c_2 s^{\nu}}{s + c_3 + 2Tc_2 s^{\nu}} \tag{52}$$

where $0 < \nu = d/2 - 1 < 1$ and

$$\begin{aligned} c_1 &= 1 + u\Delta S_d k_m^{d-2}/(d-2) \\ c_2 &= uS_d B(2 - d/2, d/2 - 1)/2^{d/2} \\ c_3 &= 2r - 2uTS_d k_m^{d-2}/(d-2). \end{aligned} \tag{53}$$

S_d is simply the angular factor from the momentum integral in (51) and $B(x,y)$ is the beta function. Investigation of $d > 4$ is straightforward, but different ranges of d

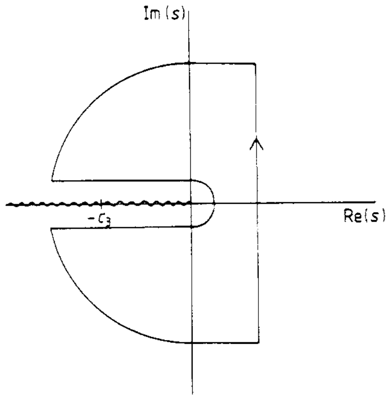

Figure 6. Contour of integration, in the complex s-plane, used to evaluate the Bromwich integral for $\eta(t)$. For $T = 0$ there is a pole on the cut at $s = -c_3$. The contribution to $\eta(t)$ from the pole is suppressed by a factor $\exp(-c_3 t)$ and is therefore negligible for large t.

($4 < d < 6$, $6 < d < 8$, etc) must be separately treated and will not be considered explicitly here.

Now that we have the explicit form for $\tilde{\eta}(s)$ we may obtain $\eta(t)$ by evaluating the appropriate Bromwich integral. Notice that since ν is not an integer there is a branch cut in the complex s-plane along the negative real axis. We close the contour in the left-half plane and deform it around the cut as illustrated in figure 6. It is easy to show that there are no poles within the integration contour for $c_3 > 0$. (Note that $c_3 = 0$ defines the critical temperature, so $c_3 > 0$ is simply the condition to be in the low-temperature phase.) Writing $s = r\exp(\mathrm{i}\theta)$, the real and imaginary parts of the denominator in (52) become $R = c_3 + r\cos(\theta) + 2Tc_2 r^{\nu}\cos(\nu\theta)$ and $I = r\sin(\theta) + 2Tc_2 r^{\nu}\sin(\nu\theta)$ respectively. But $\sin(\theta)$ and $\sin(\nu\theta)$ have the same sign for $-\pi < \arg(s) < \pi$ (since $0 < \nu < 1$), so I does not vanish within the contour of integration except for $\theta = 0$. But $R > 0$ for $\theta = 0$, since $c_3 > 0$. Hence there are no poles within the contour, and the complete contour integral vanishes. The non-zero contributions as the contour is taken out to infinity come from the original Bromwich contour and the integral along both sides of the branch cut. Thus the desired Bromwich integral is obtained from the integral of the discontinuity across the cut. This gives (with the use of (53))

$$\eta(t) = \frac{2c_2\sin(\nu\pi)}{\pi c_3^2 t^{d/2}}(T + r\Delta)\Gamma(\nu+1). \tag{54}$$

The main feature is that $\eta(t) \sim t^{-d/2}$. In other words the form of the response function is unchanged in the presence of thermal fluctuations since

$$G_{\boldsymbol{k}}(t) = \eta(t)^{-1/2}\exp(-k^2 t) = t^{d/4}h(k^2 t)$$

as found for zero temperature (see section 2).

Thermal fluctuations for $T < T_c$ are thus irrelevant to the asymptotic dynamics of the evolving system and their contribution is primarily limited to a renormalization of amplitudes. The leading correction-to-scaling associated with T can be also be obtained from the Bromwich integral. For large t, the leading correction to (54) is down by a factor of order T/t^{ν}, i.e. $T/L(t)^{2\nu}$. This can also be seen from direct inspection of (52). Thus the correction-to-scaling exponent associated with temperature is $2\nu = d-2$, in agreement with elementary arguments [7].

Equation (52) also enables us to obtain the solution *at* the critical point, by setting $c_3 = 0$. Then the dominant small-s behaviour is $\tilde{\eta}(s) \simeq (c_1/2Tc_2)s^{-\nu}$, implying a large-$t$ behaviour $\eta(t) \sim t^{-(1-\nu)}$, as may be verified by explicit evaluation of the Bromwich integral for this case. Thus, at the critical point, $G_k(t) \sim t^{(4-d)/4}\exp(-k^2 t)$, in agreement with previous results [4, 5].

Finally, $T > T_c$ implies $c_3 < 0$. Then there is single pole, on the positive real axis, within the integration contour (the Bromwich contour must, of course, be to the right of this pole). This leads to exponential growth of $\eta(t)$ and, therefore, exponential decay of the response function, i.e. a finite relaxation time. The same behaviour is obtained at any non-zero temperature for $d \leq 2$. For $d < 2$, (52) still holds, but with (taking $k_m \to \infty$) $c_1 = 1$, $c_3 = 2r$ and $c_2 = -uS_d B(d/2-1, d/2)/2^{d/2}$. Since c_2 is now negative, $\tilde{\eta}(s)$ again has a pole on the positive real axis, leading to exponential decay of the response function. Hence T is a *relevant* perturbation, as expected, for $d < 2$.

In summary, we have shown that temperature is an irrelevant variable in the ordered phase, with correction-to-scaling exponent $d-2$. This is as expected on general grounds [7], but it is nice to see it emerging cleanly from the model.

5. Long-range initial correlations

In this section we shall analyse the effect of long-range initial conditions. One physically realizable situation in which this is relevant is a quench from the critical point into the low-temperature phase. The role of long-range initial conditions in quenches to the critical point itself has been discussed elsewhere [5].

As before we shall concentrate on the case of a quench to zero temperature for convenience. We expect the results to be valid for any temperature within the ordered phase. This was shown to be the case for short-range initial conditions in the analysis of section 4. The main result of this section is that the exponent λ governing the dynamics of both the response function and the two-time correlation function does not pick up any corrections at $O(1/n)$ for any finite σ, where σ parametrizes the long-range initial correlations via

$$[\phi^i_{\boldsymbol{k}}(0)\phi^j_{-\boldsymbol{k}'}(0)] = \frac{\Delta}{k^\sigma}\delta_{i,j}\delta_{\boldsymbol{k},\boldsymbol{k}'} \qquad 0 < \sigma < d. \tag{55}$$

In the context of a $1/n$ expansion this would imply that the exponent λ is discontinuous in the limit of $\sigma \to 0$. However a renormalization group analysis [13] reveals that in fact there is a smooth crossover in behaviour at a critical value of σ which is finite and positive. In fact this critical value of sigma is given by

$$\sigma_c = d - 2\lambda_{SR} \tag{56}$$

where λ_{SR} is the value of λ calculated to $O(1/n)$ in section 3. In this section we shall find that in the region of $\sigma > \sigma_c$ (where long-range correlations are relevant) λ has the value

$$\lambda_{LR} = (d-\sigma)/2. \tag{57}$$

The calculation of the response function etc follows an identical path to the calculation described in sections 2 and 3. The only additional difficulty is that most momentum sums to be evaluated will include fractional powers of momentum in the summand. These can be evaluated at the expense of introducing an auxiliary integral. Therefore we use the identity

$$\sum_{\boldsymbol{p}} |\, \boldsymbol{p}\, |^{-\sigma} \exp(-p^2) = (\Gamma(\sigma/2))^{-1} \int_0^\infty \mathrm{d}w\, w^{\sigma/2-1} \sum_{\boldsymbol{p}} \exp\{-p^2(1+w)\}. \tag{58}$$

So following the details of section 2 we obtain the following results in the limit of $n \to \infty$:

$$G_{\boldsymbol{k}}(t) = (t/t_0)^{(d-\sigma)/4} \exp(-k^2 t) \tag{59}$$

$$C_{\boldsymbol{k}}(t,t') = \Delta k^{-\sigma}\, (tt'/t_0^2)^{(d-\sigma)/4} \exp\{-k^2(t+t')\} \tag{60}$$

and

$$S_{\boldsymbol{k}}(t) = \Delta k^{-\sigma}\, (t/t_0)^{(d-\sigma)/2} \exp(-2k^2 t). \tag{61}$$

The short-time cut-off t_0 is given by

$$t_0^{(d-\sigma)/2} = u\Delta K_{d,\sigma}/r \tag{62}$$

where

$$K_{d,\sigma} = \frac{B(\sigma/2,(d-\sigma)/2)}{2^{(d-\sigma)/2}(4\pi)^{d/2}\Gamma(\sigma/2)}. \tag{63}$$

To extend these results to $\mathrm{O}(1/n)$ we need to calculate the diagrams shown in figures 2 and 3. This time the circle has an extra momentum-dependent weight of $k^{-\sigma}$ and the single lines emerging from a circle are given by (59) above. A single line connecting two different times may be calculated as in section 3 and is again given by the ratio of the response functions at the two times. The wavy line also needs to be calculated again now that we have long-range initial conditions. It turns out that σ first appears in the quadratic part of $\rho_{\boldsymbol{k}}(t,t')$ (see section 3) and since we take $\rho = 1$ in the asymptotic regime we see that the form of $v_{\boldsymbol{k}}(t)$ is unchanged for σ non-zero.

We shall not present detailed functions for the $1/n$ contributions for this case. It is sufficient to say that the calculation of such terms follows exactly the same route as in section 3. We shall discuss the main results of the $1/n$ expansion for this case though. For the short-range initial condition (SRIC) case in section 3 we found that all diagrams had a leading-order logarithmic contribution. In terms of the response function this led to a surviving logarithm (from figure 2e) which was the source of the $1/n$ contribution to λ—the new exponent governing the dynamics of the response function and the two-time correlation function. In the long-range initial condition (LRIC) case figure 2(a)–(2d) all give logarithms and they cancel mutually as in SRIC. However the diagram 2(e) *does not* give a logarithm for finite σ. This immediately implies that the $1/n$ corrections (and presumably all further corrections) to the leading order response function (see (59)) are trivial in that they simply change the form of the scaling function. So we have

$$G_{\boldsymbol{k}}(t) = (t/t_0)^{(d-\sigma)/4} h(k^2 t). \tag{64}$$

From the definition of λ this implies the relation given in (57). As mentioned above this might lead one to suspect that the exponent λ is discontinuous in the limit of $\sigma \to 0$. This is not the case. An RG calculation [13] reveals that there is a critical value of σ such that for $\sigma < \sigma_c$ the system is dominated by the short-range fixed point and $\lambda = \lambda_{SR}$ given in (2). For $\sigma > \sigma_c$ the long-range behaviour dominates and λ acquires the value given in (57). The value of σ_c is such that the crossover in behaviour between the short-range and long-range regimes is continuous.

Evaluation of the structure factor produces the result given in (61) for $n \to \infty$. However the evaluation of the $1/n$ terms to $S_{\boldsymbol{k}}(t)$ yields two types of correction. The first (which arises from the $1/n$ response functions) has the same form as the leading order result. The second term (which arises from the diagram shown in figure 3) however is finite in the limit of $\boldsymbol{k} \to 0$ and is of the standard short-range form. So we have

$$S_{\boldsymbol{k}}(t) = (\Delta/k^{\sigma})(t/t_0)^{(d-\sigma)/2} g_1(k^2 t) + (\Delta/n)(t/t_0)^{d/2} g_2(k^2 t) + \mathrm{O}(1/n^2) \tag{65}$$

where $g_1(x)$ and $g_2(x)$ are finite in the limit of $x \to 0$. Therefore the structure factor has both short-range and long-range contributions. Working to all powers of $1/n$ will then yield a form for the structure factor composed of these two types of contribution, i.e.

$$S_{\boldsymbol{k}}(t) = (\Delta/k^{\sigma})(t/t_0)^{(d-\sigma)/2} \tilde{g}_1(k^2 t) + \Delta(t/t_0)^{d/2} \tilde{g}_2(k^2 t) \tag{66}$$

where again the two scaling functions are finite in the limit $k^2 t \to 0$. Note that if we recast (66) into the standard scaling form (10) then in the limit $k^2 t \to 0$ the structure factor will be dominated by the first term on the RHS of (66)—the long-range contribution.

6. Conclusions

In this paper we have studied in some detail the dynamics of the n-component Ginzburg–Landau model following a quench from the high-temperature to the low-temperature phase. Our analysis is centred around the $1/n$ expansion and we have restricted our attention to the case of model A dynamics (i.e. non-conserved order parameter). One of the overriding reasons for studying the $1/n$ expansion for this model is the lack of any other small parameter in which to develop a perturbation expansion. This is in contrast to critical phenomena where one may develop an expansion about the upper critical dimension. It is of interest that $1/n$ expansions at the critical point usually give rather poor results whereas the results we have obtained in this paper to $\mathrm{O}(1/n)$ are in surprisingly good agreement with results obtained from numerical simulations (for $d = 1$ and 2) [9, 12]. for values of $n = 3, 4$ and 5.

Without question the most interesting result obtained from this calculation is the new exponent that arises in the description of correlations between fields *at different times*. We have presented a new scaling form for the two-time correlation function (in equation (1)) and also the form of the new exponent λ to $\mathrm{O}(1/n)$ (in equation (2)). The structure factor presents no new surprises at $\mathrm{O}(1/n)$ in the sense that it retains the standard scaling form (10) and also equals the square of the equilibrium magnetization when summed over all momenta. As mentioned briefly in the introduction the standard scaling form for the structure factor fails for the case of a system with model B dynamics quenched into the low-temperature phase (at least in the limit of $n \to \infty$), as shown recently by Coniglio and Zannetti [6]. In that case one observes 'multiscaling' in the asymptotic regime. This means there are *two* length scales characterising the dynamics of the system which differ marginally (logarithmically).

The $1/n$ expansion was performed for the case of a quench to zero temperature for convenience. We expect the functional form of the physical quantities calculated to remain valid for quenches to any temperature less than T_c since the zero temperature fixed point controls the entire ordered phase (i.e. $T < T_c$). This was explicitly demonstrated in section 4 where we showed that the response function at finite temperature in the limit of $n \to \infty$ retained its zero temperature form.

In general one may envisage a 'triangle' of possible quenches. The case of a quench from the high-temperature phase to the critical point has been studied in detail recently [4, 5]. The main body of this paper is concerned with a quench from the high-temperature to the low-temperature phase. One can 'complete the triangle' by considering the case of a quench from the critical point into the low-temperature phase. This implies that there are long-range (power-law) correlations built into the initial state. This case was studied (again using a $1/n$ expansion) in section 5. It was found that a crossover occurs between short-range and long-range behaviour at a critical value σ_c of the parameter σ that characterizes the nature of the initial correlations. We stressed in section 5 that the $1/n$ expansion has to be augmented by a RG-type analysis [13] to establish the true nature of this crossover, i.e. to determine the value of σ_c.

Acknowledgments

TJN thanks the SERC for financial support.

Appendix

In this appendix we present the functions Λ_i and Υ referred to in section 3.

The functions in (35) and (36) are given by

$$\Lambda_1(t) = \int_{t_0/t}^{1} \frac{\mathrm{d}y}{y} \int_0^{1/y} \mathrm{d}x \left(\frac{x}{(1+x)^2(1-\alpha y)}\right)^{d/2} \left(\frac{d(d+2)}{9(1+x)^2(1-\alpha y)^2} + \frac{d^2(1+x^2)}{3x(1+x)^2(1-\alpha y)} + \frac{d(d+2)}{(1+x)^2}\right) \tag{A1}$$

where $\alpha = \alpha(x) = (1+x)/3$;

$$\Lambda_2(t) = 3^{d/2} \int_{t_0/t}^{1} \frac{\mathrm{d}y}{y} \int_0^{1} \mathrm{d}x \left(\frac{x}{(1+x)(3-x-\beta y)}\right)^{d/2} \left(\frac{d(d+2)}{(3-x-\beta y)^2} + \frac{d^2(1+x^2)}{x(1+x)(3-x-\beta y)} + \frac{d(d+2)}{(1+x)^2}\right) \tag{A2}$$

where $\beta = \beta(x) = (1-x)^2$;

$$\Lambda_3(t, k^2t) = 3^{d/2}\int_{t_0/t}^{1}\frac{\mathrm{d}y}{y}\int_0^1 \mathrm{d}x\left(\frac{x}{(1+x)(3-x)}\right)^{d/2}\left(\frac{d(d+2)}{(3-x)^2} + \frac{d^2(1+x^2)}{x(1+x)(3-x)} + \frac{d(d+2)}{(1+x)^2} + \frac{4(k^2t)(1-x)^2y}{(3-x)^3} \times\left(2(d+2) + \frac{d(1+x^2)(3-x)}{x(1+x)}\right) + \frac{16(k^2t)^2(1-x)^4y^2}{(3-x)^4}\right)\exp\left(\frac{2(k^2t)(1-x)^2y}{(3-x)}\right). \qquad \text{(A3)}$$

Finally

$$\Upsilon(t', k^2t') = \int_{t_0/t'}^{1}\frac{\mathrm{d}y}{y}\int_0^{t/yt'}\mathrm{d}x\left(\frac{x}{(1+x)^2}\right)^{d/2}\left[\frac{10d(d+2)}{9(1+x)^2} + \frac{d^2(1+x^2)}{3x(1+x)^2} + \frac{4(k^2t')y}{27(1+x)}\left(2(d+2) + \frac{3d(1+x^2)}{x}\right) + \frac{16(k^2t')^2y^2}{81}\right]\exp\left(\frac{2(k^2t')(1+x)y}{3}\right) \qquad \text{(A4)}$$

where $t \gg t' \gg t_0$.

References

[1] Gunton J D, San Miguel M and Sahni P S 1983 *Phase Transitions and Critical Phenomena* vol 8, ed C Domb and J L Lebowitz (New York: Academic) p 267
Furukawa H 1985 *Adv. Phys.* **34** 703
Binder K 1987 *Rep. Prog. Phys.* **50** 783
[2] Newman T J and Bray A J 1990 *J. Phys. A: Math. Gen* **23** L279
[3] Furukawa H 1989 *J. Phys. Soc. Japan* **58** 216; 1989 *Phys. Rev.* B **40** 2341
[4] Janssen H K, Schaub B and Schmittmann B 1989 *Z. Phys.* **73** 539
[5] Bray A J 1989 *Preprint* Manchester University
[6] Coniglio A and Zannetti M 1989 *Europhys. Lett.* **10** 575
[7] Bray A J 1989 *Phys. Rev. Lett.* **62** 2841; 1990 *Phys. Rev.* B **40** 6724
[8] Mazenko G F and Zannetti M 1985 *Phys. Rev.* B **32** 4565
[9] Newman T J, Bray A J and Moore M A 1990 *Phys. Rev.* B to be published
[10] Abramowitz M and Stegun I A (ed) 1965 *Handbook of Mathematical Functions* (New York: Dover)
[11] Ma S-K 1976 *Modern Theory of Critical Phenomena* (New York: Benjamin)
[12] Bray A J and Humayun K unpublished
[13] Bray A J and T J Newman 1990 *Preprint* Manchester University no MC-TH 90/14